



Rewarding Learning

General Certificate of Secondary Education  
2022

Centre Number

--	--	--	--	--

Candidate Number

--	--	--	--	--

## Further Mathematics

Unit 1 (With calculator)

Pure Mathematics



[GFM11]

\*GFM11\*

**WEDNESDAY 1 JUNE, MORNING**

### TIME

2 hours.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

**You must answer the questions in the spaces provided.**

**Do not write outside the boxed area on each page.**

Complete in black ink only. **Do not write with a gel pen.**

All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.

Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.

Answer **all thirteen** questions.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 100.

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

You may use a calculator.

The Formula Sheet is on page 2.

13063



\*32GFM1101\*

## Formula Sheet

### PURE MATHEMATICS

Quadratic equations: If  $ax^2 + bx + c = 0$  ( $a \neq 0$ )

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Differentiation: If  $y = ax^n$  then  $\frac{dy}{dx} = nax^{n-1}$

Integration:  $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$  ( $n \neq -1$ )

Logarithms: If  $a^x = n$  then  $x = \log_a n$

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

Matrices:

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{then } \det \mathbf{A} = ad - bc$$

$$\text{and } \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (ad - bc \neq 0)$$



1 Find  $\frac{dy}{dx}$  if  $y = 8x^4 + \frac{4}{5x^2} + 3$

Answer \_\_\_\_\_ [3]

2 Find  $\int \left( 2x^4 - \frac{1}{x^3} + 1 \right) dx$

Answer \_\_\_\_\_ [4]

[Turn over



3 (a) Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} -4 & 1 & -7 \\ -2 & -2 & 0 \end{bmatrix}$$

Calculate

(i) **B + C**

Answer \_\_\_\_\_ [2]

(ii) **AB**

Answer \_\_\_\_\_ [2]



(b) Matrices **L** and **M** are defined by

$$\mathbf{L} = \begin{bmatrix} 5 & -3 \\ -1 & -4 \end{bmatrix} \text{ and } \mathbf{M} = \begin{bmatrix} 2 & -6 \\ 5 & 3 \end{bmatrix}$$

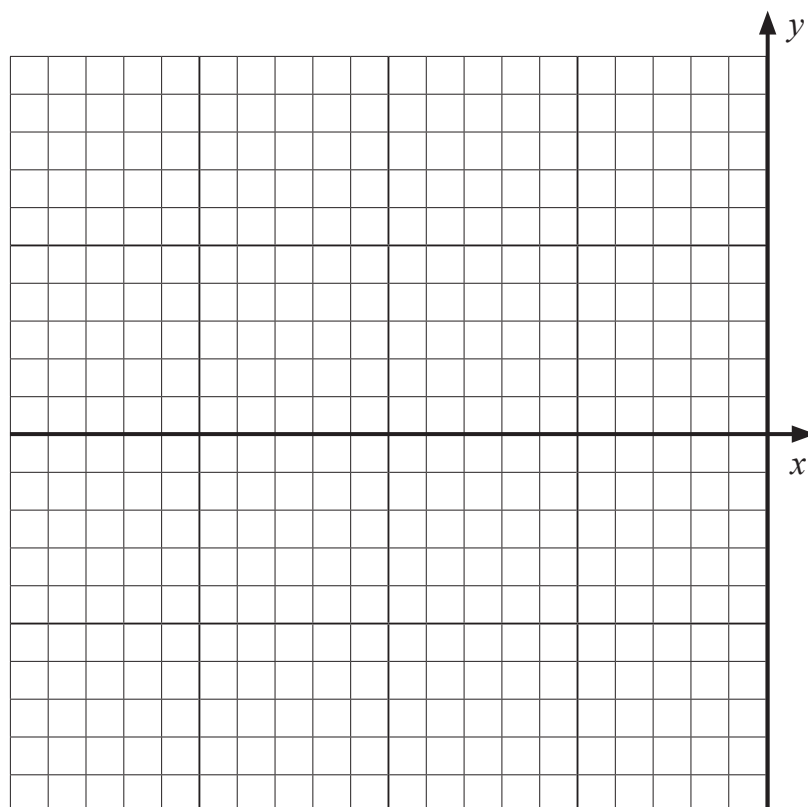
Find the matrix **X** such that  $\mathbf{LX} = \mathbf{M}$ .

Answer \_\_\_\_\_ [4]

[Turn over



4 (a) Sketch the graph of  $y = \sin x$  for  $-360^\circ \leq x \leq 0^\circ$  on the axes below.



[1]



(b) (i) Solve the equation

$$5 \sin x - 4 = 0$$

for  $-360^\circ \leq x \leq 0^\circ$

Answer \_\_\_\_\_ [3]

Q4 continues on page 8

[Turn over



(ii) Hence solve the equation

$$5 \sin (2\theta + 8^\circ) - 4 = 0$$

for  $-180^\circ \leq \theta \leq 0^\circ$

Answer \_\_\_\_\_ [2]





**BLANK PAGE**  
**DO NOT WRITE ON THIS PAGE**  
**(Questions continue overleaf)**

**[Turn over**

13063



\*32GFM1109\*

5 (a) If  $p = \log_4 x$  and  $q = \log_4 y$  write  $\log_4(4x^2\sqrt{y})$  in terms of  $p$  and  $q$ .

Answer \_\_\_\_\_ [3]



(b) Solve the equation

$$7(3-2x) = 54x$$

Answer \_\_\_\_\_ [4]



6 The sum of a number  $x$  and its reciprocal is 8

(i) Show clearly that  $x^2 - 8x + 1 = 0$

[2]

(ii) Hence, **using the method of completing the square**, solve the equation

$x^2 - 8x + 1 = 0$  to find the values of  $x$ .

Give your answer in the form  $a \pm \sqrt{b}$ .

Answer \_\_\_\_\_ [4]



7 Solve the inequality

$$12 - 4x - 5x^2 \geq 0$$

You **must** show clearly each stage of your solution.

Answer \_\_\_\_\_ [4]

[Turn over

13063



\*32GFM1113\*

8 A curve is defined by the equation  $y = 2x^2$

(i) Find the equation of the tangent to the curve at the point where  $x = 3$

Answer \_\_\_\_\_ [4]



At a point **A** on the curve, the gradient of the **normal** is  $\frac{1}{8}$

(ii) Find the coordinates of **A**.

Answer \_\_\_\_\_ [3]

[Turn over



9 (a) Expand and simplify the expression

$$(2x + 3)^3 - (6x - 1)^2$$

Answer \_\_\_\_\_ [5]



(b) Express

$$\frac{4x^2 - 1}{x + 1} \div \frac{2x^2 + x - 1}{5x + 5}$$

as a single fraction in its **simplest** form.

Answer \_\_\_\_\_ [5]

[Turn over



10 A curve is defined by the equation

$$y = x^3 + 3x^2 - 20$$

- (i) Find the coordinates of the turning points of the curve and, using calculus, identify each turning point as either a maximum or a minimum point.

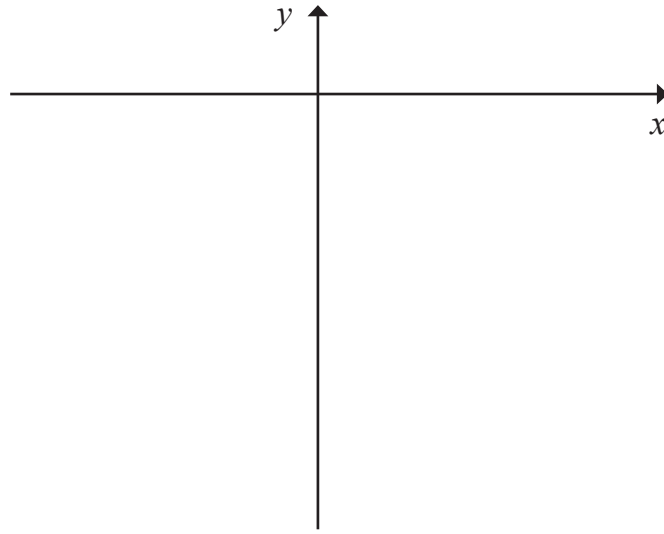
You **must** show working to justify your answer.

Answer \_\_\_\_\_ [7]



The curve crosses the  $x$ -axis once only, at the point  $(2, 0)$ .

(ii) Sketch the curve on the axes below.



[2]

(iii) Hence find the area enclosed by the curve, the  $y$ -axis and the **positive**  $x$ -axis.

Answer \_\_\_\_\_ [4]

[Turn over



11 A car rental company has Economy, Standard and Luxury cars.

Every rental involves a daily hire charge, a fuel charge per mile and a daily insurance charge.

For an Economy car:

- $x$  represents the daily hire charge, in pounds,
- $y$  represents the fuel charge per mile, in pounds,
- $z$  represents the daily insurance charge, in pounds.

It costs £150 to hire an Economy car for 5 days, travelling 600 miles.

(i) Show that  $x$ ,  $y$  and  $z$  satisfy the equation

$$x + 120y + z = 30$$

[1]



For a Standard car:

the daily hire charge is £8 dearer than that for an Economy car,  
the fuel charge per mile is the same,  
the daily insurance charge is 50% dearer than that for an Economy car.

It costs £344 to hire a Standard car for 8 days, travelling 1240 miles.

(ii) Show that  $x$ ,  $y$  and  $z$  also satisfy the equation

$$2x + 310y + 3z = 70$$

[2]

For a Luxury car:

the daily hire charge is 3 times that for an Economy car,  
the fuel charge per mile is 50% greater than that for an Economy car,  
the daily insurance charge is 2.5 times greater than that for an Economy car.

It costs £119 to hire a Luxury car for 2 days, travelling 150 miles.

(iii) Show that  $x$ ,  $y$  and  $z$  also satisfy the equation

$$6x + 225y + 5z = 119$$

[2]

Q11 continues on page 22

[Turn over



(iv) Solve the equations

$$x + 120y + z = 30$$

$$2x + 310y + 3z = 70$$

$$6x + 225y + 5z = 119$$

to find the values of  $x$ ,  $y$  and  $z$ .

Show clearly each stage of your solution.





12 Willow recorded the mass  $M$  (kg) and the metabolic rate  $R$  (watts) of 5 mammals.

The results are given in the table below.

Mammal	Mass $M$ (kg)	Metabolic rate $R$ (watts)		
Cat (Maine Coon)	6.5	2.15		
Dog (Bichon Frise)	8	2.49		
Deer (Roe Deer)	18	4.39		
Donkey (Poitou Donkey)	220	25.30		
Pony (Polo Pony)	450	41.75		

Willow believes that a relationship of the form

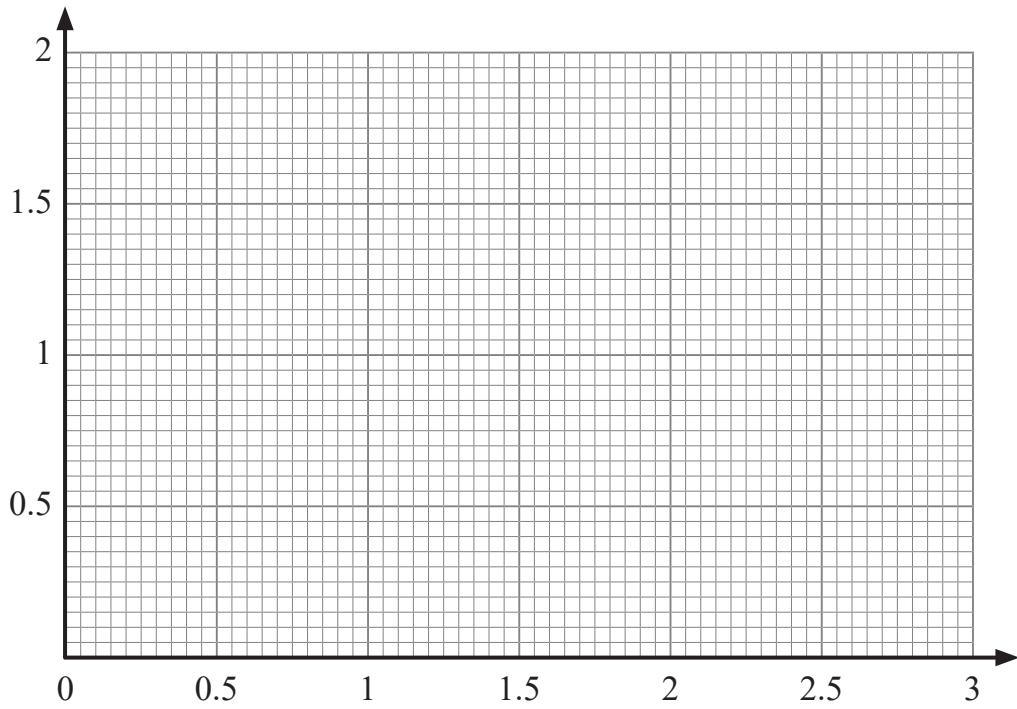
$$R = aM^n$$

exists, where  $a$  and  $n$  are constants.

- (i) Verify that a relationship of the form  $R = aM^n$  exists by drawing a suitable straight line graph on the grid opposite. Label the axes clearly.

Show clearly the values used, correct to 3 decimal places, in the table above.





[6]

Q12 continues on page 26

[Turn over

13063



\*32GFM1125\*

(ii) Hence find the values of  $a$  and  $n$ .

Answer  $a =$  \_\_\_\_\_,  $n =$  \_\_\_\_\_ [4]

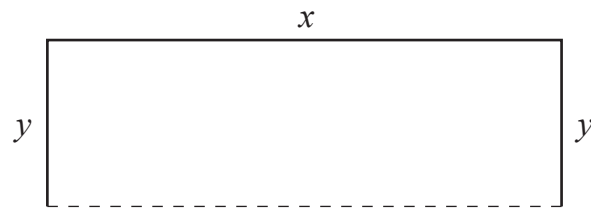
The metabolic rate of a cheetah is 9.12 watts.

(iii) Given that the same relationship in part (i) holds for a cheetah, find the mass of a cheetah.

Answer \_\_\_\_\_ kg [2]



- 13 A jeweller designs a titanium earring in the shape of an open rectangle with three sides, as shown in the diagram below.



Two sides are of length  $y$  mm and the third side is of length  $x$  mm.

The total area of the rectangle must be  $450 \text{ mm}^2$

- (i) Express  $y$  in terms of  $x$

Answer  $y =$  \_\_\_\_\_ [1]

Q13 continues on page 28

[Turn over



- (ii) Show that the expression for the length,  $L$  mm, of titanium used in the earring can be written as

$$L = x + \frac{900}{x}$$

[1]



(iii) Hence, using calculus, find the minimum length of titanium that can be used, showing that it is a minimum.

Answer \_\_\_\_\_ mm [5]

---

**THIS IS THE END OF THE QUESTION PAPER**

---



**BLANK PAGE**  
**DO NOT WRITE ON THIS PAGE**

13063



\*32GFM1130\*





**BLANK PAGE**  
**DO NOT WRITE ON THIS PAGE**

13063



\*32GFM1131\*

**DO NOT WRITE ON THIS PAGE**

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

<b>Total Marks</b>	
--------------------	--

Examiner Number

Permission to reproduce all copyright material has been applied for.  
In some cases, efforts to contact copyright holders may have been unsuccessful and CCEA will be happy to rectify any omissions of acknowledgement in future if notified.

13063/5



\*32GFM1132\*